CN510 Assignment 3: The Outstar Network

Due: October 29, 2007

The purpose of this assignment is to simulate an outstar network with three border cells. The equations for such a network can be summarized as follows:

$$dx_0 / dt = -A_0 x_0 + I_0$$
$$dx_i / dt = -Ax_i + B[x_0(t-\tau) - \Gamma]^+ z_{0i} + I_i$$
$$dz_{0i} / dt = -Cz_{0i} + D[x_0(t-\tau) - \Gamma]^+ x_i$$

where x_0 is the activity of the source cell, the x_i are the activities of the border cells (i = 1, 2, 3), and the z_{0i} are LTM traces. The constants A_0, A, B, C, D, Γ, and τ are network parameters restricted to nonnegative values. The signal I_0 can be thought of as the conditioning stimulus, and the signals I_i can be thought of as the unconditioned stimuli. The source cell sampling signal $S_0(t) = [x_0(t-\tau) - \Gamma]^+$ is defined with the threshold linear function $[x]^+ = \max(x, 0)$ and is delayed by τ seconds before reaching the border cells. Pattern variables can be defined as:

$$\theta_i = I_i / \sum_{k=1}^{3} I_k$$
$$X_i = x_i / \sum_{k=1}^{3} x_k$$
$$Z_{0i} = z_{0i} / \sum_{k=1}^{3} z_{0k} \, .$$

Note: The parameters of the three border cells are all the same; thus, this is an *unbiased* network.

(a) Let A_0 = 1, A = 5, B = C = D = 1, Γ = 0.2, and τ = 0.05. Assume that the outstar is initialized as follows: $x_0(0) = 0$, $x_i(0) = \{0.6, 0.1, 0.3\}$ and $z_{0i}(0) = \{0.7, 0.2, 0.1\}$. Assume that for $t \geq 0$, the stimulus pattern I_i = {0.1, 0.7, 0.2} is received by the border cells, and for $t \geq 2$ a conditioning signal $I_0 = 1$ is received by the source cell, as shown in Figure 2.1. Analytically determine the values of $X_i(t)$ and $Z_{0i}(t)$ as $t \to \infty$.

Hints for part (a): (1) Note that the first equation is uncoupled from the others, and that it has a simple solution as $t \to \infty$ since I_0 is constant for $t \geq 2$. (2) Also, since $x_0(\infty)$ is known, $S_0(\infty)$ is known. (3) Since the I_i are constant for $t \geq 0$, the equations for dx_i / dt and dz_{0i} / dt can be written as a (nonhomogenous, autonomous) system of linear ODEs after S_0 becomes positive. (4) All eigenvalues of this system have negative real parts, which implies that $dx_i / dt = dz_{0i} / dt = 0$ as $t \to \infty$. Alternatively, you may simply apply the Outstar Learning Theorem.

(b) Numerically integrate the outstar equations from $t = 0$ to $t = 10$ with $\Delta t = 0.05$ and the same parameters as given in part (a). Plot the pattern variables $\theta_i(t)$, $X_i(t)$, and $Z_{0i}(t)$ from $t = 0$ to $t = 10$ (make three plots, one for each i). How do X_i and Z_{0i} compare with the stimulus pattern θ_i at $t = 10$? How do your numerical values of X_i and Z_{0i} at $t = 10$ compare with your analytical values of part (a) as $t \to \infty$?

(c) Change the parameter A from 5 to 0.5, and think about what effect this may have on the coupled STM-LTM system. Then rerun the simulation. Compare your results to those of parts (a) and (b). Be sure to examine both the unnormalized and the normalized (pattern) variables before summarizing your results! Discuss your results in terms of the conditions on the outstar learning theorem.

Figure 2.1. Time dependence of stimulus patterns.