Assignment 4
Professor: Eric Schwartz – EC500

Problem 1 Assume a cell is 25μm in diameter and has a Specific Capacitance C_M of 1μF/cm^2 Assume that the intracellular concentration of $NaCl$ is 40 mM and the extra-cellular concentration of $NaCl$ is 400mM, and that initially there is no voltage across the membrane of the cell, and no ion channels that are open.

1. How many Na ions are inside the cell?
2. How many Cl ion are inside the cell? Open all Na channels and assume equilibrium is established. What is the equilibirum potential.
3. After equilibrium is established, how many sodium and chloride ions are "in" the cell. Describe where they are — is there overall charge neutrality inside the cell?
4. How many Na ions are required to flow through the channel to maintain the Nernst potential you calculated, given the capacitance of a cell of this size.
5. Calculate the ratio of the sodium ions that flowed into the cell compared to the total number that were there originally.
6. How does the final equilibrium voltage depend on the number of channels per μm that were opened?

Problem 2 In typical mammalian cells, the internal concentration of Ca^{+2} is 1mM intracellular, but only 10^{-4}mM is "free", the rest is buffered so it can't move across the membrane. The extracellular concentration of Ca^{+2} is 5mM.

1. What is the expected Nernst pontential for Ca^{+2}?

Problem 3 The Nernst-Planck equation is $J_s = -D_s(\nabla c_s + z_s c_s \frac{F}{RT} \nabla V)$, where J_S is molar current density in $\frac{\text{moles}}{\text{cm}^2 sec}$, z_S is the valence of the ion, c_S is concentration iof the ion in $\frac{mole}{\text{centimetre}^3}$, D_s the diffusion constant of the ion in $\frac{\text{cm}^2}{\text{sec}}$ which can be estimated from first principles (Kinetic Theory), (in rough agreement with its measured value) as about $10^{-5}\frac{\text{cm}^2}{\text{sec}}$, and R,F and T are the gas constant, the Faraday constant $96500\frac{\text{C}}{\text{mol}}$ and temperature in Kelvin.

1. Express the drift velocity v_s as a flux for a solution of concentration c_s of ions of valence z_s. This flux would have units $\frac{\text{coulomb}}{\text{cm}^2 sec}$, so just multiply v_s by $z_s F$. Do this and show that it is dimensionally correct.
2. The Einstein relation for mobility and diffusion constant for a single ion S is $D_s = \mu_s \frac{kT}{q}$, where k is Boltzmans constant. T is temperature in degrees Kelvin, q is an elementary charge (e.g. of an electron= 1 6 10^{-19}C), and μ_S is the mobility of ion S. (Note that $\frac{kT}{q}$ is equal to $\frac{RT}{F}$, 25mV at room temperature, 20°Centigrade). Since the definition of mobility is $v_{drift} = -\mu_s \nabla V$ in $\frac{\text{cm}}{sec}$, for a single ion, turn this into a current by multiplying by $z_s c_s F$. Show that this is dimensionally correct.
3. Since the definition of (specific) conductivity in electronic flux units is $J_s = -\sigma_s \nabla V$, write down a numerical estimate for specific conductivity (i.e. of a solution of, say, sodium ions) in terms of its concentration c_s. What are the units you obtain for σ_s? You can estimate μ_s from the Einstein relation for D_s, taking D_s to be appropriate for sodium ions, say $1.3 \cdot 10^{-5}$.
4. Express σ_s as a resistivity $\rho_s = \frac{1}{\sigma s}$ and discuss briefly the units you obtain. Note: the quantities we are working with here are somtimes called "specific resistance" or "resistivity" and "specific conductance" or "conductivity." We will be meeting quite a few more definitions of related quantities when we begin studying compartmental modeling(!).